HELMHOLTZ'S THEORY OF CONSONANCE

by

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Helmholtz as Music Theorist

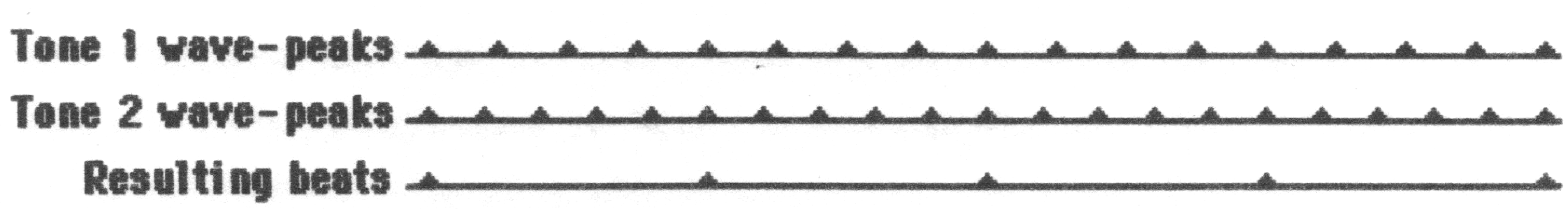
The contributions of German scientist Hermann L. F. Helmholtz (1821-1894) were wide-ranging, bringing significant advances to basic physics, thermodynamics, fluid dynamics, electromagnetism, and neuroanatomy, as well as acoustics and music theory.[1] After a decade of investigation of auditory perception, he published his treatise On the Sensations of Tone as a Physiological Basis for the Theory of Music in 1863 (last revised in 1877), explaining his many ingenious laboratory experiments in detail and relating their results to his erudite knowledge of music history in masterful fashion. Helmholtz showed that the perceived quality of a musical tone is determined by the spectral pattern of its various upper partials. He also suggested a mechanism by which the ear could function as a spectral analyzer and infer a missing fundamental from such analysis.[2] (This view of aural recognition is still widely accepted, although it is now explained psychoacoustically rather than physiologically.)[3] Finally, he developed a persuasive account of consonance, dissonance, and harmonic practice, based on the physical and physiological roots of sound.[4]

Part I of Sensations of Tone deals with the physical properties of tone as well as physiological aspects of its perception. Part II, discussed further below, develops a theory of consonance and dissonance based on auditory effects of combinatorial tones, beats, and the harmonic series. Like Rameau, Helmholtz bases his theory of consonance on scientific discoveries; unlike Rameau, however, he does not regard that theory as a full explanation of musical practice, arguing that "the essential basis of Music" is not harmony but "Melody."[5] Helmholtz distinguishes between sensory consonance (Konsonanz) and the system of tonal relationships developed in actual music (Klangverwandtschaft). In fully developed musical styles, considerations of sensory consonance doubtless influence Klangverwandtschaft; nevertheless, the latter is also to a large extent cultural, reflecting "esthetical principles" which change "with the progressive development of humanity."[6] In Part III Helmholtz examines such development, examining not only Western common practice but also more exotic traditions, such as Arabic and Persian scales which he shows to be based on "natural intonation."[7]

A Scientific View of Consonance and Dissonance

Helmholtz recognizes that Rameau was correct in seeking a natural basis for harmony and gives him (and d'Alembert) credit for a major theoretical accomplishment, given the "scanty materials" provided by the science of that time. Nevertheless, Helmholtz does not regard physical nature with the same reverence as Rameau; in reference to inharmonic partials, for instance, he remarks that Rameau "might have heard many a perfectly dissonant chord" if he had listened to a wider array of sonorous bodies. To infer harmony from nature requires a broader view of nature, also encompassing requirements of the human ear and other acoustical phenomena. Moreover, Helmholtz (unlike Rameau) brings to this investigation a thorough understanding of scientific method and a keen appreciation of the difference of viewpoint between artist and scientist.[8]

When two tones occur in the same physical space, acoustical interference occurs between their vibration patterns. If the tones are simple (without overtones) and have the same pitch, their sound waves may reinforce each other if the tones are "in phase." On the other hand, if the tones have a phase relationship of 180° and the same amplitude, they may cancel each other entirely. Now if the two tones differ slightly in pitch (i. e., frequency), then they will alternate between "in-phase" and "out-of-phase" relationships, giving rise to the periodic fluctuations in amplitude known to piano tuners as "beats" (illustrated below).[9]



The frequency of these beats is equal to the difference of the two input frequencies. If this difference is small, then the beats are slow and can be easily heard; moreover, Helmholtz observes, they are "by no means disagreeable to the ear." If their frequency is very gradually increased, they can still be perceived as beats, although they become too rapid to be counted. Even above 20-30 cps (within the range of audible tones), they may continue to be heard as beats (and not as tones), creating a psychological impression described by Helmholtz as "jarring and rough." The semitone b'-c", for example, produces 33 beats per second, which is "very jarring." Beyond this approximate point, the beats gradually become too rapid to be perceived as such and the sensation of "roughness" disappears. The 66 beats per second created by the whole tone b♭'-c" is already less harsh, while at 88 beats per second (a'-c") hardly any roughness is detectable.[10]

The perception of beats is also affected by the pitch register of the input tones. If both tones are raised by an octave, then the frequency of the beats doubles; the audibility range for beats also increases, although not proportionally. For example, Helmholtz reports that the 132 beats per second produced by the semitone b'''-c'''' remain audible, although weak. Thus the audibility limit depends both on the size of the pitch difference and on the pitch register of the constituent tones, and Helmholtz considers possible anatomical explanations for this "compound dependence." Because the dependence on register is less than proportional, however, transposing the same interval to higher octaves and thus doubling the rate of beats tends to render them less noticeable. Beats for the whole tone become difficult to sense beyond about 2000 cps (produced near the high end of a piano keyboard for that interval). Beats for major and minor thirds, which are imperceptible in mid-range, become "decidably rough in the lower octaves and produce distinct beats"; this observation, of course, agrees well with the common musical experience of "muddiness" in that register.[11]

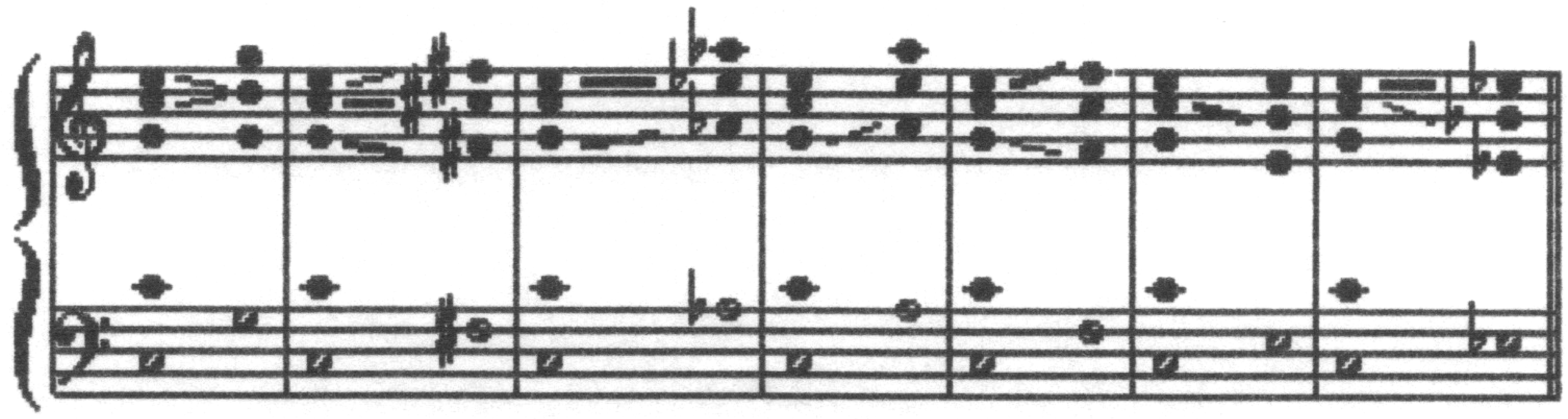
"Intermittent excitement" of "auditory nerve fibers," Helmholtz argues, produces an unpleasant or irritating effect, analogous to that of a rapidly flickering light on the eye. Thus "consonance is a continuous, dissonance an intermittent sensation of tone." In dissonances the rapid beats create roughness which is normally masked by the stronger fundamental tones but which nevertheless conveys an impression to the ear. Such dissonance-producing beats may also be created between overtones of two compound tones, or between an overtone of one and the fundamental of the other. In determining whether a given interval is consonant or dissonant, the lower partials play the largest role, in part because such partials (particularly the first six) are strongest in musical tones.[12]

A second factor involved in dissonance are "combination tones" which are often produced when two pitches are sounded simultaneously; unlike beats, these have the wave-forms of tones and are perceived as such. As Helmholtz demonstrates, both by mathematical physics and by laboratory experiments, they exist objectively and are not merely products of the ear. Two kinds of combinational tones may occur: the difference tone (previously studied by Sorge and Tartini), whose frequency is the difference of the two input frequencies; and summational tones, which are far weaker and of less importance to music theory. Both kinds may result from upper partials as well as from the input tones themselves.[13]

When two input tones produce partials (or difference tones) at adjacent frequencies, beats will arise; in middle registers these disturbances are most noticeable when the partials are separated by about a semitone (as discussed above). Using the relative intensity of partials in a violin tone as a basis, Helmholtz examines the relative strength of such disturbances for various intervals. Because beats will clearly be eliminated insofar as the two tones share identical lower partials, his search for consonances focusses on pitches which involve "coincidences" among the first eight partials.[14]

In the unison, octave, twelfth, and double octave, the higher tone merely reinforces certain partials of the lower tone, so that no new disturbances are generated at all; hence Helmholtz calls these the "absolute consonances." In fact, the strength of the fundamental and first overtone are sufficient to create strong disturbances against the minor second and major seventh, thus rendering these "the harshest dissonances." For the same reason, even the major second and minor seventh must be rated as dissonances, although somewhat milder.[15]

The perfect fifth is also found to be a "good" consonance; only minor disturbances are created here by proximity of partials (see below). The tritone, on the other hand, is seriously disturbed at both the second and third partials of its higher tone and is thus "decidedly dissonant." The minor sixth is also disturbed at both the second and third partials of



——— indicates harsh semitone disturbance
——— indicates milder whole tone disturbance

its upper tone The latter disturbance, however, involves a relatively weak fifth partial, so that the minor sixth is more consonant than the tritone; contextual musical considerations (discussed below) ensure its acceptance as a consonance. The major sixth suffers only slight disturbance and is thus clearly consonant.[16]

The perfect fourth is a marginal case; as Helmholtz notes, its consonant status is a matter of long dispute. Its upper tone is disturbed at both the second and fourth partials. The major third (as shown above) involves one disturbance, while the minor third suffers two; these all involve high partials, creating beats which will be too rapid to be audible if the intervals are placed in higher registers. Helmholtz observes that thirds have been accepted as "imperfect" consonances only since the thirteenth century. Typical musical settings of early medieval music, involving men's voices and a lower register, would have made the beats produced by such thirds clearly unpleasant.[17]

More generally, the roughness of any particular interval (other than an absolute consonance) is greater in a lower register. Helmholtz calculates the relative placement of various just-tempered intervals in such a way as to generate equivalent roughness, as shown in part below.

In effect this vertical ordering represents relative consonance or dissonance. (The minor seventh and tritone shown here use the seventh partial, which is lower than an equal-tempered b♭; these intervals are less dissonant than their equal-tempered counterparts.)[18]

Helmholtz also notes that the major third and major sixth must be tuned very precisely in order to avoid conflicts between upper partials. He therefore distinguishes these "medial" consonances from the "imperfect" consonances of the minor third and minor sixth, which involve no low coinciding partials and are thus less precisely determined.[19]

Even in the case of simple tones, which have no upper partials, beats will often arise from combination tones if the original tones are in a dissonant relationship. For example, two simple tones a minor ninth apart will generate a difference tone a semitone above the lower of the two tones; consequently, beats will still occur, creating the perception of dissonance.[20]

In the marginal case of the minor sixth, the structure of "the whole tonal system" requires that it be accepted as consonant in preference to intervals using the seventh partial, such as the "sub-minor" seventh. The inversion of the minor sixth, of course, is the very useful major third. On the other hand, "the inversion or transposition of an interval formed with the number 7 leads to intervals worse than itself." Finally, the minor sixth is preferable to the subminor seventh for purposes of scale construction.[21]

Consonant and Dissonant Chords

Clearly, in order for a chord to be consonant, all of the intervals created by its tones must also be consonant. Working under the assumption of just intonation, Helmholtz shows that only major and minor triads (and their inversions) satisfy this criterion. The augmented triad, he argues, is clearly dissonant under such intonation and is interpreted as dissonant even under equal temperament: "this chord is well adapted for shewing that the original meaning of the intervals asserts itself even with the imperfect tuning of the piano, and determines the judgment of the ear." More generally, he argues that differences of consonance and dissonance under just intonation (which he prefers) are "much more decided and conspicuous, than in the equal temperament." Yet these differences remain, in attenuated form, even under modern tuning; for example, tempering introduces only one beat per second to the perfect twelfth between d' and a" on an equal-tempered piano, thus leaving intact that interval's consonant status.[22]

Combination tones, Helmholtz observes, render the minor triad "very decidedly less harmonious" than the major. These effects, he argues, are insufficient to render the minor triad dissonant, but they still create "a sensible increase of roughness." A root-position C-minor triad in close structure, for example, generates a difference tone of A-flat a major seventeenth below its root.

> The foreign element thus introduced into the minor chord is not sufficiently distinct to destroy the harmony, but it is enough to give a mysterious, obscure effect to the musical character and meaning of these chords. . . . Hence minor

chords are especially adapted to express mysterious obscurity or harshness.[23]

This major-minor difference is again more pronounced under just intonation; Helmholtz suggests that it may have been one principal reason for the older practice of closing minor-key compositions with a Picardy third, a custom whose decline roughly coincided with the rise of equal or near-equal temperament. Helmholtz also determines those dispositions of minor and major triads which minimize the effects of combination tones, pointing out how such arrangements are preferred in works of Palestrina and Mozart.[24]

Modern Perspectives on Helmholtz

Results of recent experiments, in which listeners were asked to evaluate the degree of consonance of intervals formed by pairs of pure tones, agree quite well with Helmholtz's observations. Maximum perceived dissonance occurred at about a semitone from A-440; moreover, the size of this maximally dissonant interval decreased in higher registers (just as Helmholtz observed). Modern acousticians also agree with Helmholtz that the listener can reconstruct a fundamental from a spectrum of its harmonic overtones. It is now believed, however, that this process is at least partly mental rather than physiological; this skill is apparently acquired by the individual while learning to recognize vowels in speech.[25]

While much disagreement persists about the nature of consonance, Ernst Terhardt notes that Helmholtz's approach "must be considered even today as being most appropriate, successful, and promising."[26] Terhardt proposes a two-component approach similar to Helmholtz's distinction between Konsonanz and Klangverwandtschaft. Sensory consonance, applicable principally to isolated intervals and chords, is characterized by the "lack of annoying features of a sound," including in particular Helmholtz's "roughness." "Harmony," on the other hand, involves affinities of pitches in tonal context.[27]

Helmholtz, as discussed above, regards such harmonic relationships as largely culturally determined. Considering the common twentieth-century propensity for cultural explanations, one might expect that modern commentators would be inclined to embrace the same view. Terhardt, however, argues that harmony (like sensory consonance) has a more universal scientific basis:

> In contrast to Helmholtz's assumptions, harmony (Klangverwandtschaft) today is not considered a cultural product that evolved in parallel with music but a psychoacoustically based phenomenon that accompanies the perception of complex signals, in particular, speech.[28]

Conclusions

Helmholtz offers a sophisticated view of consonance and dissonance which highlights practical issues important to musicians.

First, he anticipates twentieth-century theorists such as Hindemith and Howard Hanson in arguing that "there is a continuous series of degrees" between consonance and dissonance. Second, he shows that consonance is determined, not only by interval size, but also by a host of other factors, often ignored in the oversimplified views of most theorists. Except for the absolute consonances, intervals tend to be less consonant in lower registers than in higher, or when realized by timbres rich in upper partials, such as those of bowed string instruments. Some intervals, such as the major third, are more agreeable in compound form (i. e., as major tenth); others, such as the perfect fourth and both sixths, become more dissonant when compounded. When three or more tones are present, factors such as combination tones may affect consonance.[29]

Helmholtz's theory is not based on numerology (like that of Pythagoras or Zarlino) or even on mere physics. He notes how Euler sought to explain consonance in terms of a psychological preference for the "perfection" manifested by ratios of small whole numbers. Euler, however, could not identify "the mode in which the mind contrives to perceive the numerical ratios of two combined tones." In Helmholtz's view, the mind perceives "only the physical effect of these ratios" and not the ratios themselves.[30] Thus he moves beyond numerology and beyond the corps sonore to examine the relationship of resonant bodies to the listening observer. That musical consonance has a natural basis was not a new idea, but Helmholtz places it on firm ground that has held up relatively well under the scrutiny of modern science.

NOTES

1. Richard M. Warren, "Helmholtz and His Continuing Influence," Music Perception 1:3 (Spring 1984): 253-255.

2. Hermann L. F. Helmholtz, On the Sensations of Tone as a Physiological Basis for the Theory of Music, trans. Alexander J. Ellis (London: Longman, Green, & Co., 1875); reprint (with an Introduction by Henry Margenau), New York: Dover Publications, 1954), Introduction and 65-119 (page references are to reprint edition).

3. Warren, 262-263. See also Ernst Terhardt, "The Concept of Musical Consonance: A Link between Music and Psychoacoustics," Music Perception 1:3 (Spring 1984): 287.

4. Helmholtz, 152-233.

5. Helmholtz, vii, 4-233.

6. Terhardt, 282-283; Helmholtz, 235.

7. Helmholtz, 280-285 and Part III passim.

8. Helmholtz, 232-233; Warren, 255-256.

9. Helmholtz, 159-165.

10. Helmholtz, 164-170.

11. Helmholtz, 171-173.

12. Helmholtz, 169-170, 182-184, 226.

13. Helmholtz, 152-161.

14. Helmholtz, 183, 186-187.

15. Helmholtz, 187-188.

16. Helmholtz, 188-189.

17. Helmholtz, 189-190.

18. Helmholtz, 191-192.

19. Helmholtz, 187, 194.

20. Helmholtz, 203-204.

21. Helmholtz, 183, 227-228.

22. Helmholtz, 180-181, 211-213, 319-320.

23. Helmholtz, 214-216.

24. Helmholtz, 217, 219-226.

25. Terhardt, 279-281, 287-292; Warren, 264-265.

26. Terhardt, 277.

27. Terhardt, 281-284.

28. Terhardt, 288.

29. Helmholtz, 189, 195-196, 204-211, 227.

30. Helmholtz, 229-231.

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