ANNOTATED TRANSLATION OF AN EXCERPT FROM RAMEAU'S *NOUVEAU SYSTÈME DE MUSIQUE THÉORIQUE*

by

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NOUVEAU SYSTEME DE MUSIQUE THEORIQUE

Translator's Note

The following translation is based on a facsimile of the original French-language publication of Jean-Philippe Rameau's 1726 treatise, Nouveau Système de Musique Théorique.[1] This excerpt, in which Rameau advocates a near-equal system of temperament, is taken from the final chapter of that work. (In his Harmonic Generation, published eleven years later, the author expresses different views, advocating true equal temperament.[2])

Translation of Chapter XXIV (pages 107-111)

On Temperament

[107] Temperament is the modification of the natural[3] ratio of an interval, without impairing the satisfaction which the ear should receive [from that interval].

Such temperament is absolutely essential in tuning[4] organs and other keyboard instruments. Practical musicians apply it quite regularly, using only the unaided ear, without having taken great pains to discover its justification. Yet they do it without purpose, if one must is to believe one of the most famous geometers[5] of the last century, when he asks:

> "Why, in a song with one or several voices, is it impossible to keep the same pitch level unless one tempers (without conscious attention) the consonances, so that they should be somewhat removed from their natural state? What none of our [terrestrial musicians] has yet explained, nor given the reason whereof, is why this temperament is in string instruments the most perfect of all, when one decreases the fifth by one quarter of a comma.[6]

We are indebted to Mr. Sauveur for establishing a system which gives all possible temperaments.[7] But it still lacks something which has also eluded other systems, namely, defining the correct temperament and founding it on persuasive arguments.

In order to be able to establish a temperament which is free of problems, we must consider three things: the results of experiments with vibrating strings, the ratios indicated by numbers, and the customary method of tuning harpsichords.

Experiments with vibrating strings instruct us that the fifth resonates even if it is tuned slightly smaller than its natural size, whereas the major third no longer resonates if it is altered even slightly. Those are facts of experience concerning which anyone can ascertain the details for himself.

Numbers, for their part, determine for us the ratio of a fifth (A to E) reduced by one comma (27 : 40) in diatonic systems, without other fifths being increased as well.[8] All other intervals are found to remain unaltered, except for the minor third from A to C (27 : 32), which naturally leads to this reduction in size of the fifth.[9] Now, these numbers agree [108] entirely with observations of vibrating strings: for if the major third cannot be altered at all without ceasing to resonate, then the entire reduction of the fifth must rest on the numbers determined by the minor third.[10]

As to the customary tuning of harpsichords, the first few fifths are customarily reduced very slightly; after the fourth interval of a fifth has been tuned, it is checked against the tone with which the tuning process began, and with which it should form a major third. Thus if one does not find at this point a major third in such purity as the ear demands, then one must begin the tuning process anew, reducing the fifths a little more: for the lack of accuracy which one hears in such cases in the major third almost always results from not having sufficiently reduced the fifths.

This custom, which is merely drawn from observations of common practice, nevertheless relates to what we have just noted in observations of vibrating strings and of numbers, just as it must logically follow.

Now when one has reached the middle of the tuning process, one begins to make the fifths somewhat truer [i. e., closer to their natural ratio (2 : 3)], and more and more true up to the last one, for reasons which will become apparent below.

This process of temperament would not be necessary at all, if one never had to pass through more than a single modulation;[11] since it should only be necessary then to follow what diatonic systems dictate. But if one is free to pass from one key to another, as we discussed in Chapter VII,[12] one should be aware of it only in passing. For example, moving from the key of C to that of A, one must have in that case two E's, of which one would have [value] 5 for the third of C and the other [value] 81 for the fifth of A, according to the order of generation of chords or progressions, Chapter III, page 24.[13]

Similar differences will follow for all the tones, both for those which one will want to use as tonics of a key and for those through which one will wish to pass.[14] Thus the attention which would then be required to choose one sound over another in each different modulation would make its execution virtually impracticable for musical instruments. And it is mainly in order to avoid this problem that it has been deemed appropriate to use but a single E, which can serve at the same time as the third of C and as the fifth of A; other tones are determined in like manner.

In order for the same sound simultaneously to serve as the third of one [tone] and the fifth of another, it was necessary to temper it in a certain way. Long experience has shown the reason for such [109] temperament, and the progressions set forth [in the Table of Progressions] are going to lead us to its discovery.

Following the progression by powers of three from C up to B#^{C}, one will find that this B#^{C}, which must give us a pitch equivalent to C on the keyboard, nevertheless exceeds it by a maximum comma.[15] Now if it were simply a question of reducing this B#^{C} to the unison or to one of the octaves of C by a proportional tempering of each of the fifths from C up to this B#^{C}, then it should only be necessary to divide the comma in question into as many equal parts as there are fifths from C up to this B#^{C}, in order then to reduce each fifth by one of these parts of the comma. But since it is absolutely essential that the fourth of these fifths should make a pure major third with the beginning pitch, such a reduction process would not suffice, and we must find other means of arriving at our objective.

Since E^{a} (81), which makes the fourth interval of a fifth after C in the first column, exceeds by a major comma the E (5) in the second column which makes the major third of this C, and since it is absolutely necessary to adjust this E from [value] 81 to [value] 5 so that it will make a true major third with C, it is only necessary to reduce each fifth by a quarter of this comma. Then this quarter-comma less for each fifth will lead to an E reduced by the entire comma, since after thus reducing the fourth fifth after C as well as the three preceding fifths, there will then be four quarters less of the comma, that is, one comma less. Hence from [value] 81 it will be reduced to [value] 80 or [equivalently][16] to [value] 5, making a sufficiently accurate fifth with A and an accurate major third with C.

By this reduction of the fifths, we comply with our observations on experiments with strings, on ratios drawn from numbers, and on the customary tuning of harpsichords. We answer at the same time Mr. Hughens's query, since we give the reason for this reduction. But there remains the question of knowing why one makes the fifths a little more accurate when one reaches the midpoint of this tuning.

If one pursued the proposed reductions up to the twelfth fifth, one would arrive then at B# (125), which is less than C by a major diesis:[17] for according to the order of the Table of Progressions, if after having reduced the E^a in the first column to the E in the second column, one continues to reduce fifths in the same manner, the G#^{a} in this second column will be reduced to the G# in the third column, and by the same ratio the B#^{a} in this third column will be reduced to the B# in the fourth column. [110] Thus instead of finding at the twelfth fifth a pitch equal to C, one would find one smaller by a major diesis: that is why it is necessary to [begin to] make the fifths a little more true when one reaches the middle of the tuning process, in order to be able to recover in the later ones what one has lost excessively in the first ones.[18]

There is only a maximum comma too much between the C and the B#^{c} in the first column, whereas there is a major comma plus a minor comma too little between the same C and the B# in the fourth column.[19] Now, having reduced each fifth by a fourth of the major comma, one is already a maximum comma plus a minor comma lower, when one has reached the G# in the third column.[20] Thus it is essential to make the fifths truer from this B# up to the end, in order to recover the extra minor comma which one has lost.[21]

It is not necessary to wait until one has reached the G# to begin making the fifths truer, and one ought to begin doing so as early as the fifth between C# and G# (assuming one began the tuning process with C) in order to have less to recover in the following fifths. If this method is used, the final major thirds will suffer much less; although one cannot then avoid making these thirds, as well as the two last fifths, a little too large.

The excess size of the two last fifths and of the four or five last major thirds is acceptable, not only because it is almost imperceptible, but also because it is found in the least utilized modulations, unless one should choose them expressly in order to create a harsher expression, etc. For it can well be noted that we receive different impressions from intervals, to the degree that they are altered differently: for example, the major third, which naturally inspires joy in us, according to our experience, arouses in us notions of fury when it is too large; and the minor third, which naturally inclines us to gentleness and tenderness, saddens us when it is too slight.

Skilled musicians know how to take advantage of these different effects of intervals and to make expressive use of this alteration, which might [otherwise] be subject to censure.

In order for intervals to preserve maximum accuracy in the most often used modulations, it is necessary to begin the tuning process with B-flat and then to make the fifths a little truer only from B to F# on.[22]

By means of such temperament, pitches which [would otherwise] differ by one or two commas are merged into a single pitch; they are always caused to sound by the same note or the same key on the harpsichord and on several other [111] instruments. Thus we do not find more than one E, or more than one B, and so forth. The notes C and B# are the same single key [on the keyboard], and so on. But then these notes or keys change names according to the different modulations through which one passes; the key which is called C in the keys of C, F, G, A, and so on will be called B# in the keys of B#, G#, E#, C#, and so forth; and the other keys are treated in like manner.

Under this temperament all the semitones are roughly either major or mean; the two mean semitones make up roughly a minor whole tone, and the mean semitone and the major semitone together make up a major whole tone.[23]

In order to find the ratios in this temperament, it would be necessary to be able to divide the comma into equal parts, which does not fall within the abilities of our system, where the divisions are harmonic, i. e., divisions into unequal parts; so that no matter how far one might carry the indicated progressions, one would never find an interval in them which makes up exactly one quarter of another interval.[24] But as one can do without [these exact ratios], whether in keyboard tuning where one perceives well enough the point to which the fifths must be reduced, or else in the manufacture of instruments where we believe it necessary to follow the ratios of [Mr. Sauveur's] most perfect system, we shall leave this matter to inquisitive persons, who will be able to satisfy themselves on this point with the help of Mr. Sauveur's system.

Example I

TABLE OF PROGRESSIONS

Column 1		**Column 2**		**Column 3**		**Column 4**		**etc.**
C	1	E	5	G♯	25	B♯	125	
G	3	B	15	D♯	75	F♯♯	375	
D	9	F♯	45	A♯	225	C♯♯	1125	
A	27	C♯	135	E♯	675	G♯♯	3375	
E^a	81	G♯^{a}	405	B♯^{a}	2025	D♯♯^{a}	10125	
B^a	243	D♯^{a}	1215	F♯♯^{a}	6075	A♯♯^{a}	30375	
F♯^{a}	729	A♯^{a}	3645	C♯♯^{a}	18225	E♯♯^{a}	91125	
C♯^{a}	2187	E♯^{a}	10935	G♯♯^{a}	54675	B♯♯^{a}	273375	
G♯^{b}	6561	B♯^{b}	32805	D♯♯^{b}	164025	F♯♯♯^{b}	820125	
D♯^{b}	19683	F♯♯^{b}	98415	A♯♯^{b}	492075	C♯♯♯^{b}	2460375	
A♯^{b}	59049	C♯♯^{b}	295245	E♯♯^{b}	1476225	G♯♯♯^{b}	7381125	
E♯^{b}	177147	G♯♯^{b}	885735	B♯♯^{b}	4428675	D♯♯♯^{b}	22143375	
B♯^{c}	531441	D♯♯^{c}	2657205	F♯♯♯^{c}	13286025	A♯♯♯^{c}	66430125	

etc.

NOTE: In the original edition of Rameau's treatise, superscripts do not appear for the entries E^a (81) and B^a (243) in Column 1. These superscripts are nevertheless required both for internal consistency and for consistency with the text of Chapter XXIV; accordingly, they have been restored here.

NOTES

1. Jean-Philippe Rameau, *Nouveau system de musique théorique* (Paris: Jean-Baptiste-Christophe Ballard, 1726; facsimile, New York: Broude Brothers, 1965).

2. See Mark Lindley, "Temperaments," in Stanley Sadie, ed., *The New Grove Dictionary of Music and Musicians* (London: Macmillan Publishers Ltd., 1980), XVIII, 669.

3. The term *juste* (and *justesse* and other derivatives), which Rameau uses repeatedly with regard to natural intervals and their mathematical ratios, is of course related to the English term "just," as in "just intonation." Nevertheless, because Rameau employs the term in a somewhat wider context, it is translated here as "natural" (or, in other contexts, as "pure," "true," or "accurate").

4. Here Rameau uses the term *partition*, which he defines in a previous chapter (Rameau, 19), as "*la manière d'accorder*" (the manner of tuning).

5. As indicated by a marginal note in the original edition, this reference is to Dutch scientist Christiaan Huygens.

6. Christiaan Huygens, *Nouveau traité de la pluralité des mondes*, trans. I. Dufour (Paris: J. Moreau, 1702), 150. Huygens speculates that a hypothetical musician from Jupiter or Venus (unlike musicians on our own planet) would perhaps be able to explain the reasons for such adjustments. He then recommends that the desired temperament could be accomplished "without any perceptible error" by dividing the octave into thirty-one equal parts - a feat which, he notes, could be accomplished only if this extraterrestrial musician were acquainted with logarithms.

7. Joseph Sauveur, *Principes d'acoustique et de musique, ou système général des intervalles des sons* (Geneva: Minkoff Reprint, 1973). Rameau cites this work in his Chapter I (Rameau, 17).

8. As Rameau indicates in a previous chapter (Rameau, 15), the term "comma" without qualification means the ratio 80 : 81. On occasion he also refers to this ratio as the *Comma majeur*. He also mentions two

other species of acoustical "comma." The Comma mineur is given by the ratio 2025 : 2048, while the Pythagorean comma or Comma maxime is given by the ratio 524288 : 531441. Each of these ratios can be derived from combinations of natural octaves, fifths, and major thirds:

$$(80/81) = (2/3)^4 / (1/2)^2 / (4/5),$$

i. e., 4 fifths - 2 octaves - major third;

$$(2025/2048) = (1/2)^3 / (4/5)^2 / (2/3)^4,$$

i. e., 3 octaves - 2 major thirds - 4 fifths;

$$(524288/531441) = (2/3)^{12} / (1/2)^7,$$

i. e., 12 fifths - 7 octaves.

By "diatonic systems" Rameau means a tuning system based on the diatonic tones in his Table of Progressions, reproduced in part in Example 1 and further explained in note 12 below. Among these tones the only perfect fifth which is altered from its natural ratio (2 : 3) is that between A and E, given by the ratio 27 : 40. (Although the table indicates value 5 for E, transposing it to the proper octave gives value 40.) This ratio is a natural fifth reduced by one comma (80 : 81), as can be determined by the calculation below.

$$(2/3) \ / \ (80/81) \ = \ (2/3) \times (81/80) \ = \ 27/40$$

9. A natural major third (ratio 4 : 5) combined with this altered minor third (27 : 32) gives a natural fifth reduced by one comma (27 : 40).

$$(4/5) \times (27/32) \ = \ 27/40$$

10. In other words, if the resonance requirements of both of these important intervals are to be realized, then the fifths must be tempered in the manner described.

11. Rameau uses the term modulation here in an older sense, defining it as "the ordering of the tones of a mode" (Rameau, 30), and using the term mode in the sense of the modern concept of "key."

12. Rameau, 37-42.

13. Rameau's table of geometric progressions has been transcribed in part in Example I. ("Ut" has here been represented by the letter "C," and so on.) This table, based on a tuning system employing combinations of natural fifths and natural major thirds, elegantly represents the ratio of each tone to the initial tone C. For example, the ratio of F-sharp to C is 45 : 1; octaves can be added by multiplying by the appropriate power of two, giving 45 : 64. (By way of comparison, an equal-tempered tritone is approximately 45.26 : 64). It will be noted, however, that this method generates an infinite series of different possible values for F-sharp (and the other notes), which Rameau differentiates by means of superscript letters. Consequently, this table illustrates graphically the complexities introduced by key-to-key modulations, complexities which in themselves necessitate moving beyond what Rameau calls "diatonic" tuning and implementing a system of temperament.

In this table, which can be extended indefinitely in both dimensions, successive rows represent progression by natural fifths, while successive columns represent progression by natural major thirds. Thus the value in row m, column n of Rameau's table is given by the formula below.

$$3^{(m-1)}5^{(n-1)}$$

14. Rameau uses the term principal for "tonic," explaining that he wishes "to convey that it is the principal object of the entire modulation" (Rameau, 30). "Modulation," again, refers to the ordering of tones within a key.

15. The ratio 524288 : 531441, as explained in note 7 above. The value 524288 is a power of two and hence gives C in the appropriate octave.

16. The values 80 and 5 are separated by 4 octaves and are treated as if they were the same value, by octave equivalence.

17. Rameau uses the term Dieze majeur, corresponding to the ratio 125 : 128. The term "diesis" is used by various authors to indicate a number of different small intervals.

18. In other words, if each of the major thirds (from C to E-sharp, from

E-sharp to G-sharp, and from G-sharp to B-sharp) is reduced by one comma, then one finds that the resulting B-sharp (125) is lower than C (1, equivalent to 128). In order to avoid this dilemma, Rameau is arguing, one should compensate by making the later fifths in this series larger (i. e., closer to their natural ratios).

19. For the latter comparison:

$$125/128 = (80/81) \times (2025/2048)$$

20. By multiplying the ratios for the various commas (see note 5 above) together, it is readily seen that a reduction of two major commas (between C and G-sharp) would be equivalent to a reduction of a maximum comma plus a minor comma.

21. In other words, one wishes only to compensate for the extra maximum comma resulting from pure fifths tuning (represented by the first column in Rameau's table). But the proposed method has already overcompensated by a minor comma when the G-sharp is reached.

22. Thus the harsh special effects of oversized fifths and thirds will be concentrated in keys which presumably are less commonly used.

23. From earlier parts of the treatise (Rameau, 3-28), one can determine the following ratios for each of the intervals mentioned in this passage.

major semitone	15 : 16
mean semitone	128 : 135
minor whole tone	9 : 10
major whole tone	8 : 9

24. Since a quarter-comma would correspond to the fourth root of the fraction (80/81), and since this root is an irrational number, one could never find the value for a quarter-comma in Rameau's table of progressions, no matter how far it might be extended.